Government General Degree College, Chapra

Department of Mathematics

Project Name- Recognize conics from general equation of second degree

Participants- Soniya Khatun, Sourav

Mentor- Mr. Biswajit Paul

Definitions of a conic:

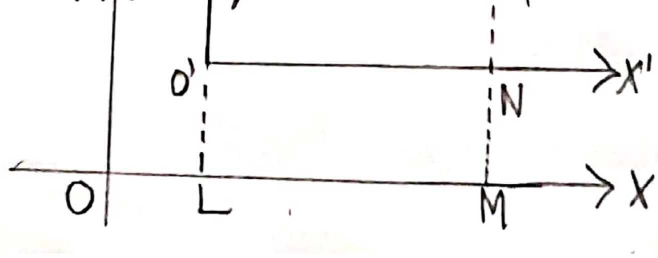
If a point moves in a plane in such a way that its distance from a fixed point always bears a constant ratio to its distance from a fixed straight line, then the locus of the moving point is called a conic sections or simply a conic.

A curve, which is represented by an equation of the second degree in cartesian co-ordinate system, is called a curve of the second order.

Consider the general equation of the second degree

$$ax^2+2hxy+by^2+2gx+2fy+c=0 \quad \ldots\ldots$$

Change of origin without changing the directions of the axes (translation):



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Let ox, oy be the set of rectangular axes referred to which the co-ordinates of an orbital point p are (x,y). Let O' the new origin, be at (h,k) and $O'x'$, $O'y'$ be the new set of axes parallel to the original axes. Let the co-ordinates of P Reffed to the new set of axes be (x',y') then,

$$x = OM = OL + LM = OL + O'N$$
$$= h + x'$$
$$y = PM = MN + PN = O'L + PN = k + y'.$$

Hence,

$$x = x' + h, \quad y = y' + k$$

are the translation formula for the translation of axes.

Thus the equation $f(x,y) = 0$ with reference to the odd set of axes becomes $f'(x'+h, y'+k) = 0$ with reference to the new set of axes Removing the primes to put in general form the new equation becomes $f(x+h, y+k) = 0$

$\therefore$ The above formula may also be written as $x' = x - h$, $y' = y - k$.

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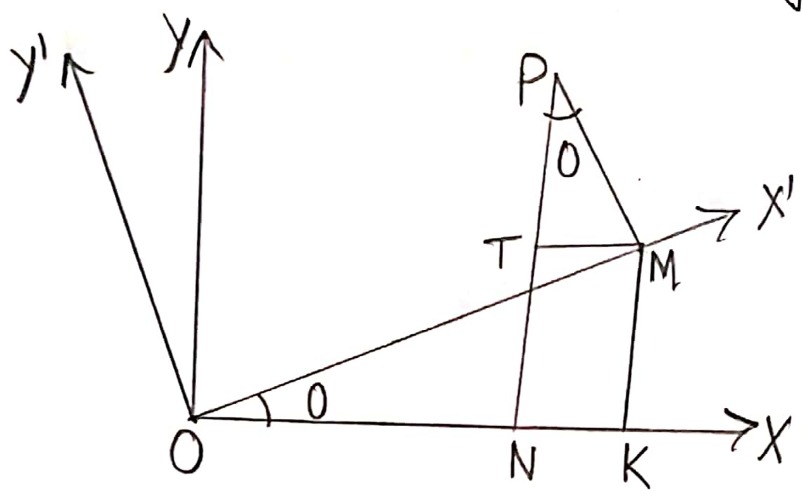
Transformation form one pair of rectangular axes to another with the same origin (Rotation)



Let the axies ox and oy be turned about O through an angle θ to the position ox' and oy'. Let P be any point (x, y) referred to the system ox, oy and (x', y') referred to the new set of axes ox', oy'.

We have $OM = x'$, $PM = y'$

then $x = ON = OK - NK = OK - MT = x'\cos\theta - y'\sin\theta$

Since,

$$\angle TPM = 90^\circ - \angle TMP = \angle TMO = \theta.$$

Again, $y = PN = TN + PT = PT = x'\sin\theta + y'\cos\theta$

Hence,

$$x = x'\cos\theta - y'\sin\theta, \quad y = x'\sin\theta + y'\cos\theta \ldots$$

are the transformation formula for the rotation of axes.

Nature of The Conic :

The general equation of The second degree in x and y is...

$$ax^2+2hxy+by^2+2gx+2fy+c=0 \quad \ldots (1)$$

This represents a second order curve and hence a conic.

Let us introduce The notation

$$\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = abc+2fgh-af^2-bg^2-ch^2$$

and $D = \begin{vmatrix} a & h \\ h & b \end{vmatrix} = ab-h^2$

Δ is called The discriminant of and is invariant under translation and of axes.

If $\Delta=0$, then The question (1) represents a pair of straigh lines.

If $\Delta=0$, then The equation (1) represent a circle.

Let $S(\alpha,\beta)$ be the focus $lx+my+n=0$ be The directrix and $P(x,y)$ be a point on The conic whose, then by definition of conic.

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D	Δ	Canonical form	Nature
$D>0$	$\Delta<0$	$\frac{x^2}{A^2}+\frac{y^2}{B^2}=1$	Elipse
$D>0$	$\Delta>0$	$\frac{x^2}{A^2}+\frac{y^2}{B^2}=-1$	Imaginary Elipse
$D<0$	$\Delta<0$	$\frac{x^2}{A^2}-\frac{y^2}{B^2}=-1$	Hyperbola
$D<0$	$\Delta>0$	$\frac{x^2}{A^2}-\frac{y^2}{B^2}=1$	Hyperbola
$D>0$	$\Delta=0$	$Ax^2+By^2=1$	Elipse or pair of imaginary strai line
$D\neq0$	$\Delta=0$	$y^2-K^2x^2=0$	pair of interesting straigh line
$D=0$	$\Delta=0$	$y^2=m^2(\neq0)$	parallel
$D=0$	$\Delta=0$	$y^2=0, x^2=0$	coincide st. line